Modeling and Control of a Bergey-Type furling Wind Turbine

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Objectives

- To study the overspeed-protection furling mechanism of the Bergey Wind Turbine and
  - Model the furling mechanism (for real-time)
  - Effect of aerodynamics, generator, electrical side
- Build a simulation package
- To design an active yaw mechanism for larger turbines (and improve performance)
  - Control Objectives
  - Control Laws and Algorithms
  - Actuation mechanism

Schedule And Status:

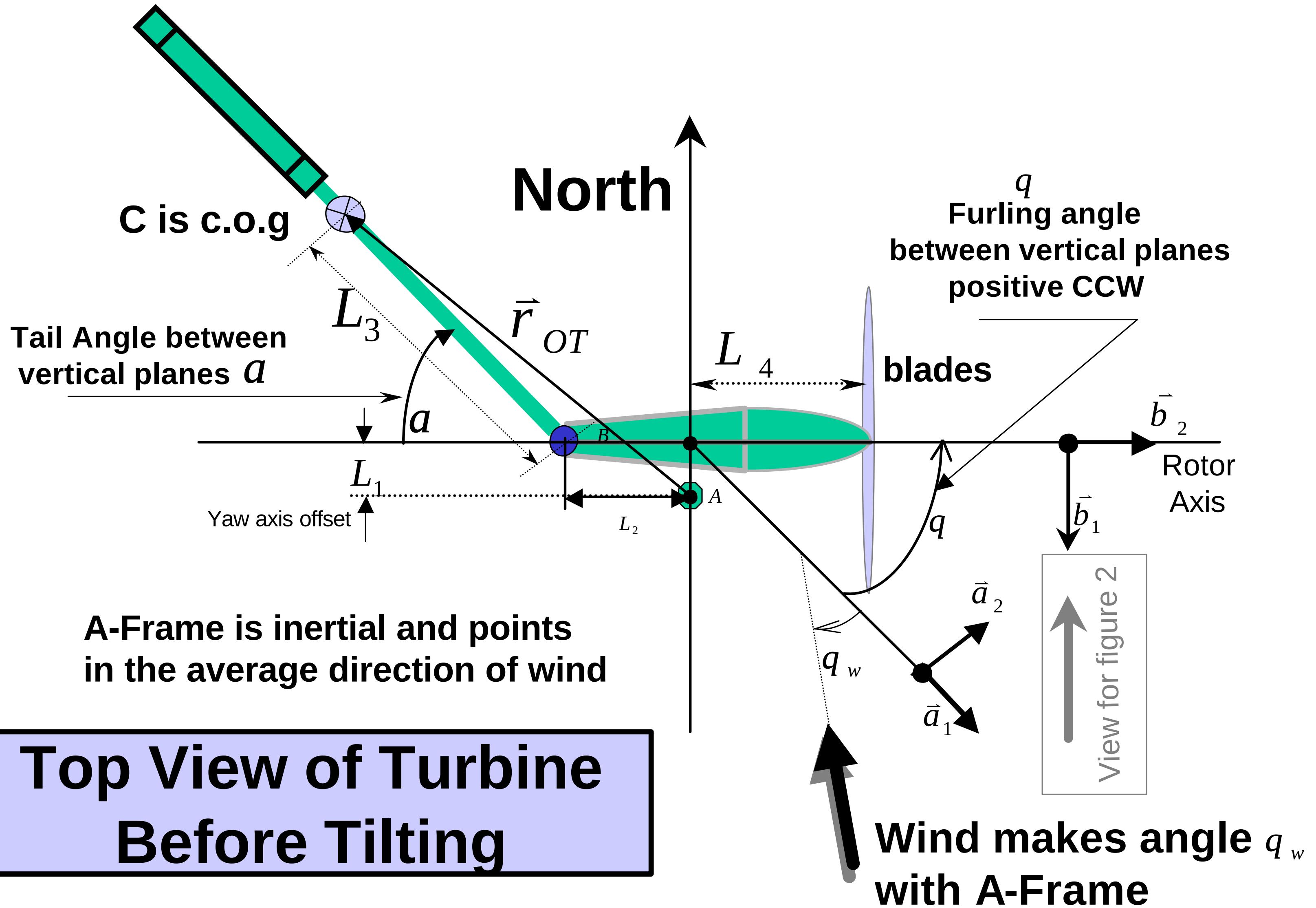
- Completed the Lagrangian derivation of EOM of furling mechanism
- Analyzed YawDyn Output
- Approximated YawDyn data using fuzzy models (real-time; derivatives for linearization)
- Analyzed steady-state behavior of turbine
- Studying actuation choices for active mechanism
- Concurrently writing software in MATLAB and Simulink
- Designing control laws

Bergey Wind Turbine

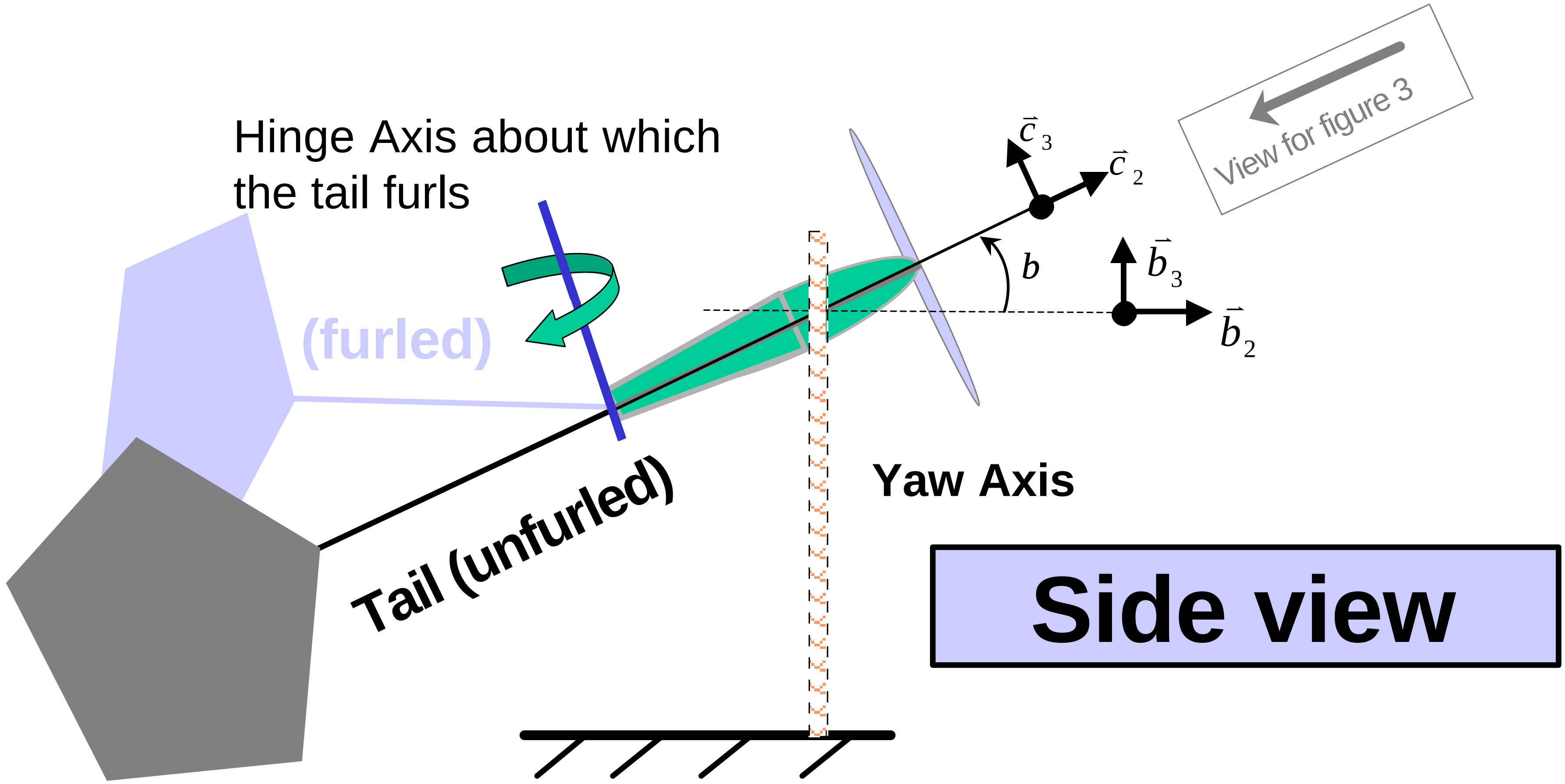
- Bergey Excel 10KW Turbine
- Operates in the boundary layer of the Earth (turbulence)
- Tail always “points” in the average direction of the wind
- Uses a passive auto-furling mechanism to protect turbine/generator combination from overspeeding in high winds
- Low maintenance/fail safe
- New 40KW turbine envisaged: Is a (still fail-safe) autofurling mechanism desired?
- Use furling to achieve protection AND optimum performance.

Specifications of the 10 KW Bergey Wind Turbine

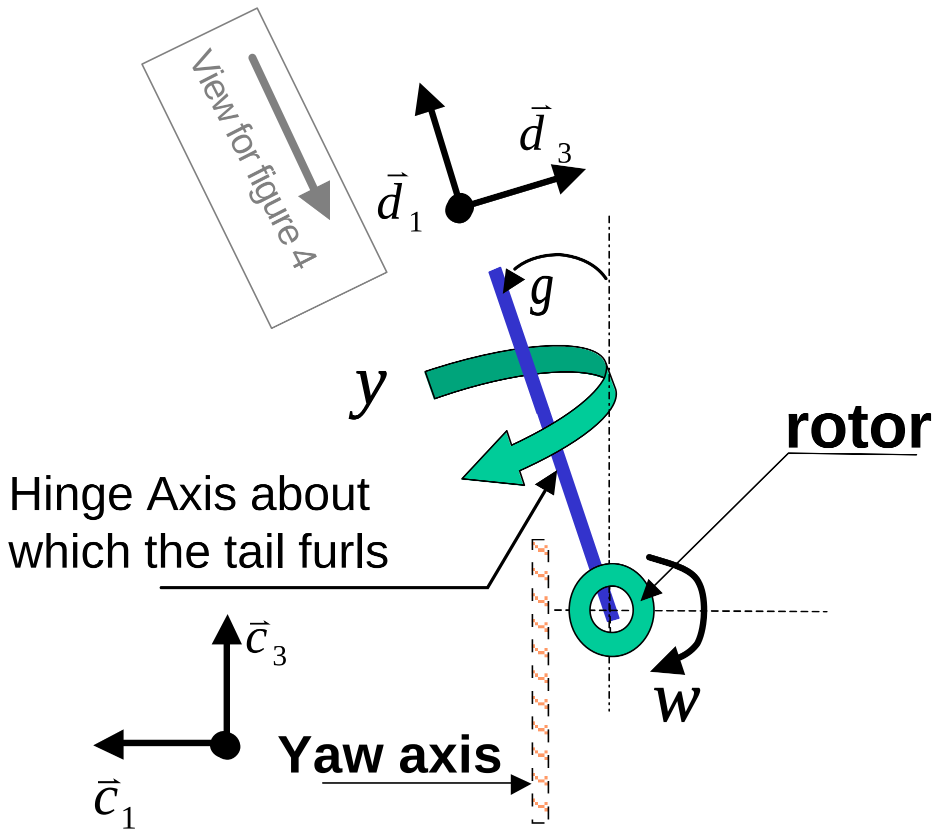
Start-up Wind Speed: 3.4 m/s (7.5 mph)
Cut-in Wind Speed: 3.1 m/s (7 mph)
Rated Wind Speed: 13 m/s (29 mph)
Rated Power: 10,000 Watts
Cut-Out Wind Speed: None
Furling Wind Speed: 15.6 m/s (35 mph)
Max. Design Wind Speed: 53.6 m/s (120 mph)
Type: 3 Blade Upwind
Rotor Diameter: 7 m (23 ft.)
Blade Pitch Control: POWERFLEX®
Overspeed Protection: AUTOFURL
Drive: Direct
Temperature Range: -40 to +140 Deg. F
Generator: Permanent Magnet Alternator
Output Form: 3 Phase AC, Variable Frequency (Regulated 48 - 240 VDC after VCS-10 or 240 VAC, 1Ø, 60 Hz with Powersync® inverter)



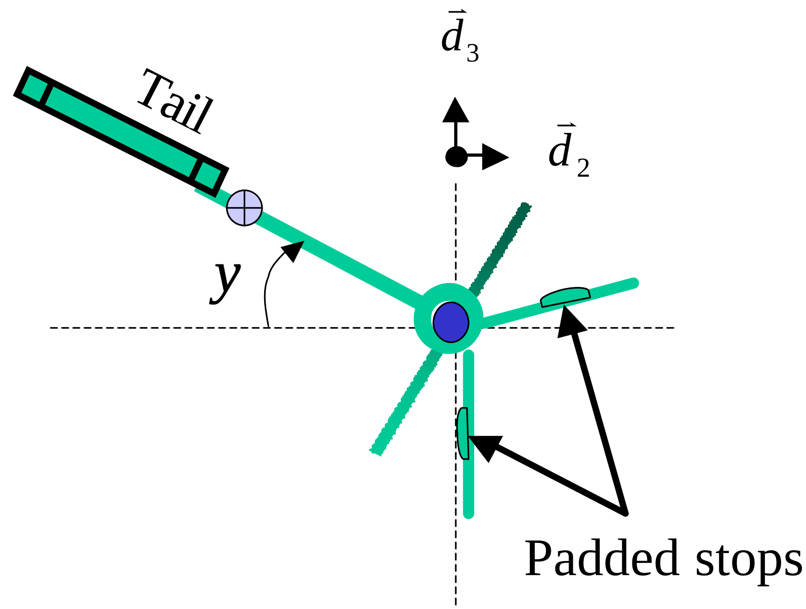
Top View of Turbine Before Tilting


Hinge Axis about which the tail furls
(furled)
Tail (unfurled)
Yaw Axis
View for figure 3
$\vec{c}_3$
$\vec{c}_2$
b
$\vec{b}_3$
$\vec{b}_2$
Side view

Other Views



Front View Along Rotor



Top View Along Hinge Axis

Notation

- J_{GY} : Moment of inertia of generator about the yaw axis after tilting.
- J_{OT} : Moment of inertia of tail about an axis passing by its center of gravity.
- M_G : Mass of the generator.
- M_T : Mass of the tail.
- L_1 : Distance between the yaw axis and the rotor axis.
- L_2 : Distance along the rotor between the yaw axis and the tail hinge.

Notation (Continue)

- L_3 : Distance between the hinge and the center of gravity of the tail.
- L_4 : Distance between yaw axis and the plane of the rotation of the blades.
- L_{ac} Distance between the yaw axis and the aerodynamic center of the tail.
- θ : Furling angle from the A frame to a vertical plane containing the rotor axis (positive counter clockwise).
- θ_w : Angle from the A frame to the wind direction (positive counter clockwise).

Notation (Continue)

- $\Delta\theta = \theta - \theta_w$: Angle from wind to rotor (positive counter clockwise).
- $\Delta\theta - \psi$: Angle from the tail to the wind (positive clockwise).
- β : Tilt angle between the rotor axis and the horizontal.
- α : Angle between two vertical planes, one containing the tail and the other containing the rotor axis.
- γ : Angle of rotation around the rotor of the tail hinge axis measured from a vertical plane
- ψ : Angle of rotation of the tail about the tail hinge

Coordinate Frames

- Inertial frame $F_A = (\vec{a}_1, \vec{a}_2, \vec{a}_3)$ point in the average direction of the wind.
- F_B frame of reference attached to nacelle before tilting

$$\vec{b}_1 = \vec{a}_1 \cos\theta + \vec{a}_2 \sin\theta \tag{1}$$

$$\vec{b}_2 = -\vec{a}_1 \sin\theta + \vec{a}_2 \cos\theta \tag{2}$$

$$\vec{b}_3 = \vec{a}_3 \quad \text{(vertical upward)} \tag{3}$$

- $F_C = (\vec{c}_1, \vec{c}_2, \vec{c}_3)$ after tilting
- $F_D = (\vec{d}_1, \vec{d}_2, \vec{d}_3)$ aligned with the tail hinge

Drag and Lift on Tail

$$\vec{a}_D = -\vec{a}_2 \cos\theta_w + \vec{a}_1 \sin\theta_w$$ drag parallel to wind

$$\vec{a}_L = -\vec{a}_1 \cos\theta_w - \vec{a}_2 \sin\theta_w$$ ...

- $\vec{F}_{tail} = F_{drag}\vec{a}_D + F_{lift}\vec{a}_L$ where

$$F_{drag} = \frac{1}{2}\rho V_{wind}^2 A_{tail} C_D, \quad C_D = constant$$ assumption

$$F_{lift} = \frac{1}{2}\rho V_{wind}^2 A_{tail} C_L, \quad C_L = C_L^0 \sin(\Delta\theta - \psi)$$ assumption

- $\Delta\theta - \psi =$ "apparent" angle of attack on tail.

- $\vec{F}_{tail} = \begin{bmatrix} F_4 & F_5 & F_6 \end{bmatrix} \{d\}$

$$F_4 = F_{drag}(-\sin\gamma\sin\Delta\theta + \cos\gamma\sin\beta\cos\Delta\theta)$$
$$- F_{lift}(\sin\gamma\cos\Delta\theta + \cos\gamma\sin\beta\sin\Delta\theta),$$ etc ...

Potential and Kinetic Energies

$$V = -M_T g(L_2 \sin\beta + L_3 \cos\psi \sin\beta - L_3 \sin\psi \sin\gamma \cos\beta)$$
$$T^* = \frac{1}{2} M_T \vec{v}_T \cdot \vec{v}_T + \frac{1}{2} J_{OT} \vec{\omega}^T_A \cdot \vec{\omega}^T_A + \frac{1}{2} J_{GY} \dot{\theta}^2$$

$$\vec{r}_{OT} = -L_1 \vec{b}_1 - L_2 \vec{c}_2 - L_3 \cos\psi \vec{d}_2 + L_3 \sin\psi \vec{d}_3$$

From yaw axis to aerodynamic center of tail

$$\vec{v}_T = (L_2 \cos\beta + L_3 \cos\psi \cos\beta + L_3 \sin\psi \sin\gamma \sin\beta)\dot{\theta}\vec{b}_1$$
$$-(L_1 + L_3 \sin\psi \cos\gamma)\dot{\theta}\vec{b}_2 + L_3\dot{\psi} \sin\psi \vec{d}_2 + L_3\dot{\psi} \cos\psi \vec{d}_3$$

$$\vec{\omega}^T_A = \vec{\omega}^T_D + \vec{\omega}^B_A = -\dot{\psi}\vec{d}_1 + \dot{\theta}\vec{b}_3$$

Rotational speed of tail with respect to inertial A Frame

Kinetic Energy

$$\boxed{T^* = \tfrac{1}{2}J_1(\psi)\dot{\theta}^2 + \tfrac{1}{2}J_2\dot{\psi}^2 + J_{12}(\psi)\dot{\theta}\dot{\psi}}$$

$$J_1(\psi) = M_T[(L_2\cos\beta + L_3\cos\psi\cos\beta + L_3\sin\psi\sin\gamma\sin\beta)^2$$

$$-(L_1 + L_3\sin\psi\cos\gamma)^2] + J_{OT} + J_{GY}$$

$$J_2 = M_T L_3^2 + J_{OT}$$

$$J_{12}(\psi) = M_T L_3[(L_1 + L_3\sin\psi\cos\gamma)(-\sin\psi\cos\beta + \cos\psi\sin\beta\sin\gamma)$$

$$-\cos\psi\cos\gamma(L_2\cos\beta + L_3\cos\psi\cos\beta$$

$$+L_3\sin\psi\sin\beta\sin\gamma)] - J_{OT}\cos\beta\cos\gamma$$

Lagrange's Equation of Motion

Lagrange's equations of motion are

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial \dot{\theta}}\right) - \frac{\partial T^*}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q_\theta$$
$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial \dot{\psi}}\right) - \frac{\partial T^*}{\partial \psi} + \frac{\partial V}{\partial \psi} = Q_\psi$$

where Q_θ and Q_ψ represent generalized forces (torques) arising from aerodynamic forces, friction, etc...

$$\delta W = Q_\theta \delta\theta + Q_\psi \delta\psi. \tag{6}$$

Assume $\delta\theta = 0$ when finding Q_ψ...

Generalized Force for Furling Angle

$$\boxed{\delta\theta = 0}$$

$$Q_\psi = \underbrace{M_{stop}}_{\text{When tail leans against the stops}} + \underbrace{M^{\psi}_{Tail}}_{\text{Aerodynamic furl moment due to tail}} + \underbrace{M^{\psi}_{Control}}_{\text{Control moment}}$$

$$M_{stop} = \begin{cases} K_s\psi & \text{if} \quad \psi < 0 \\ 0 & \text{if} \quad 0 < \psi < 70^\circ \\ -K_s(\psi - 70^\circ) & \text{if} \quad \psi > 70^\circ \end{cases}$$

assumption

$$M^{\psi}_{Tail} = -\left(\vec{r}_{hac} \times \vec{F}_{Tail}\right) \cdot \vec{d}_1, \quad r_{hac} = \begin{cases} \text{between tail hinge and} \\ \text{aerodyanimc center of tail} \end{cases}$$

Tail Aerodynamic Furling Moment

$$M^{\psi}_{tail} = -(\vec{r}_{hac} \times \vec{F}_{tail}).\vec{d}_1 = -\det([r_{hac}, F_{tail}, d]) = \det([F, r_{hac}, d])$$

$$= \boxed{F_{drag}L^{\psi}_{drag} + F_{Lift}L^{\psi}_{Lift}}$$

$$L^{\psi}_{drag} = L_{ac}[\cos\Delta\theta(\cos\psi\sin\gamma\sin\beta - \sin\psi\cos\beta) + \sin\Delta\theta\cos\psi\cos\gamma]$$

$$L^{\psi}_{Lift} = L_{ac}[\sin\Delta\theta(\sin\psi\cos\beta - \cos\psi\sin\gamma\sin\beta) + \cos\Delta\theta\cos\psi\cos\gamma]$$

- If $\sin\gamma\sin\beta \approx 0$

assumption

$$L^{\psi}_{drag} \approx L_{ac}(\sin\Delta\theta\cos\psi\cos\gamma - \cos\Delta\theta\sin\psi\cos\beta) \qquad (7)$$

$$L^{\psi}_{Lift} \approx L_{ac}(\sin\Delta\theta\sin\psi\cos\beta + \cos\Delta\theta\cos\psi\cos\gamma) \qquad (8)$$

- If $\cos\beta = \cos\gamma$, we have

$$L^{\psi}_{drag} = L_{ac}\sin(\Delta\theta - \psi)\cos\beta$$

$$L^{\psi}_{Lift} = L_{ac}\cos(\Delta\theta - \psi)\cos\beta$$

Depends on the angle between tail and wind

Generalized Moment Q_θ with $\delta\psi = 0$

$$Q_\theta = \underbrace{M_{YawDyn}}_{\substack{\text{Total aerodynamic}\\ \text{yaw moment}\\ \text{via YawDyn}}} + \underbrace{M^{\theta}_{Tail}}_{\substack{\text{Aerodynamic}\\ \text{moment due}\\ \text{to forces on tail}}} + \underbrace{M^{\theta}_{Control}}_{\substack{\text{Control}\\ \text{moment}}}$$

assumption

$$M_{YawDyn} = \underbrace{M_{Thrust}}_{\substack{\text{Rotor thrust } \times\\ \text{yaw axis}\\ \text{offset } (L_1)}} + \underbrace{M_{Nacelle}}_{\substack{\text{Aerodynamic}\\ \text{moment on}\\ \text{nacelle body}}} + \underbrace{M_{Lateral}}_{\substack{\text{Lateral force}\\ \times \text{ distance to}\\ \text{yaw axis}}}$$

Tail Aerodynamic Yaw Moment

$$M_{tail}^{\theta} = (\vec{r}_{Oac} \times \vec{F}_{Tail}) \cdot \vec{b}_3 = \det([r_{Oac}, F, b])$$
$$= \boxed{F_{drag} L_{drag}^{\theta} - F_{lift} L_{lift}^{\theta}}$$

assumption

Simplifying $\sin\gamma \sin\beta \approx 0$,**and** $\cos\beta = \cos\gamma \approx 1$

$$L_{drag}^{\theta} = L_1 \cos\Delta\theta - L_2 \sin\Delta\theta - L_{ac} \sin(\Delta\theta - \psi)$$

$$L_{lift}^{\theta} = L_1 \sin\Delta\theta + L_2 \cos\Delta\theta + L_{ac} \cos(\Delta\theta - \psi)$$

Depends on the angle between tail and wind as well as yaw angle

Tail Aerodynamic Moments

$$M_{tail}^{\psi} \approx \frac{1}{2}\rho V_{wind}^2 A_{tail} L_{ac} \cos\beta [C_L \cos(\Delta\theta - \psi) + C_D \sin(\Delta\theta - \psi)]$$

$$M_{tail}^{\theta} = \frac{1}{2}\rho V_{wind}^2 A_{tail} \{[C_D L_1 - C_L L_2] \cos\Delta\theta - [C_D L_2 + C_L L_1] \sin\Delta\theta$$
$$- L_{ac} [C_D \sin(\Delta\theta - \psi) + C_L \cos(\Delta\theta - \psi)]\}$$

Equations of Motion For Furling Mechanism

$$J_1(\psi)\ddot{\theta} + J_1'(\psi)\dot{\psi}\dot{\theta} + J_{12}'(\psi)\dot{\psi}^2 + J_{12}(\psi)\ddot{\psi} + b_1\dot{\theta} = Q_\theta \qquad (16)$$

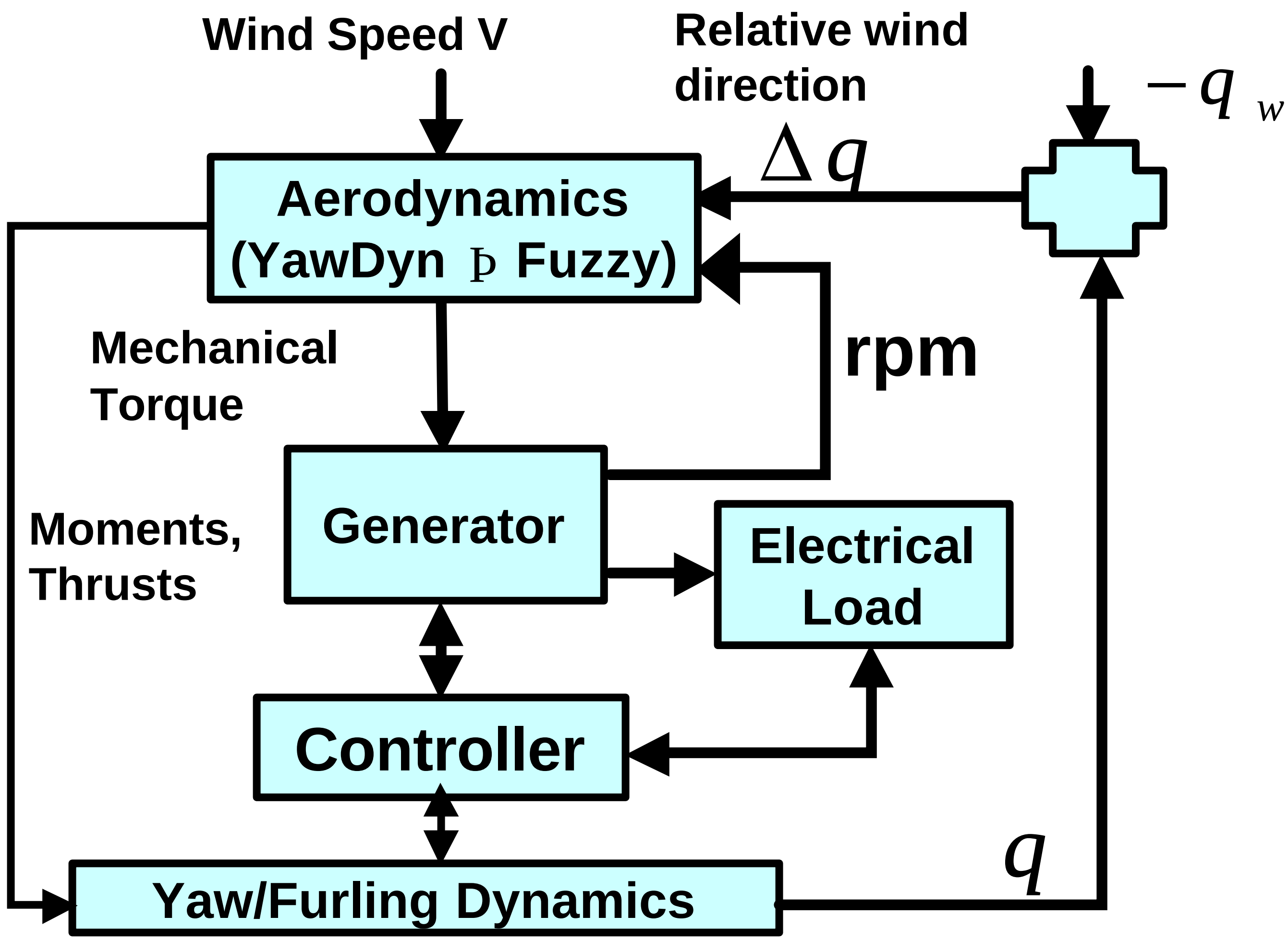
$$J_2\ddot{\psi} + J_{12}'(\psi)\dot{\theta}\dot{\psi} + J_{12}(\psi)\ddot{\theta} + \frac{\partial V}{\partial \psi} + b_2\dot{\psi} = Q_\psi \qquad (17)$$

where b_1 and b_2 are added friction coefficients

assumption

Generator and electrical load dynamics are coupled through the dependence of Q_θ on the rotor rpm.

General Wind Turbine Model



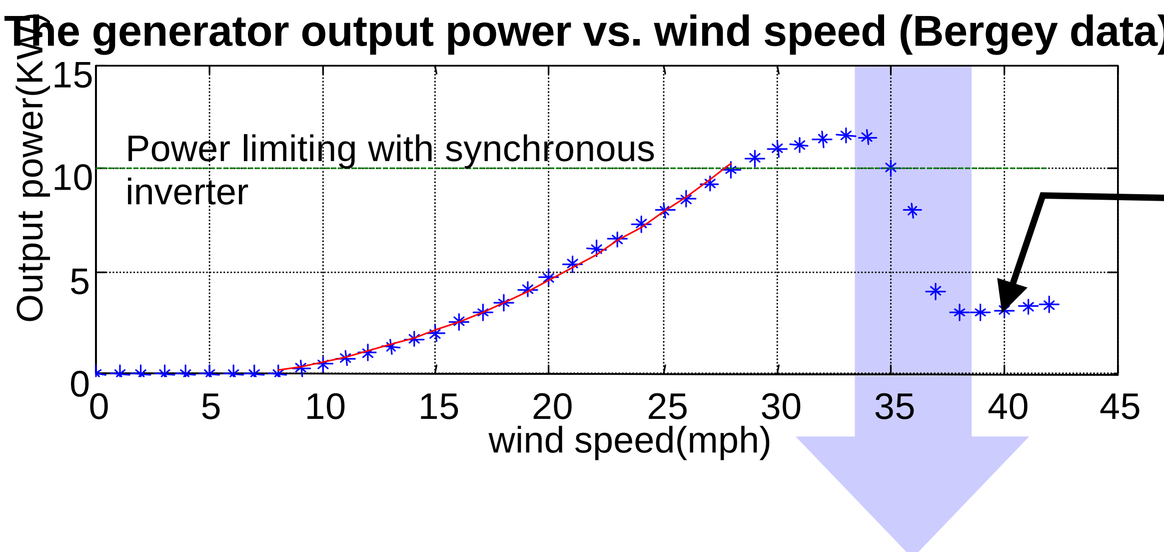
Building the Simulation Software

1. Use the Lagrangian to derive the basic equations of motion of furling mechanism
2. Run YawDyn to characterize the turbine power output and the corresponding aerodynamic forces on turbine (as a function of wind speed, rpm, and angle between wind and rotor
3. Obtain fuzzy approximation
4. Add friction and stop terms
5. Add generator (Simulink) and electrical load/power electronics model (Simulink? Sabre?)
6. Add/modify dynamics for actuator
7. Add model for disturbances (turbulence?)

Generator Model

- Intersection of turbine Torque-Speed characteristics with that of generator yield equilibrium point at onset of furling
- This curve depends on the electrical load seen by the generator
  - Resistive load (interesting?)
  - Battery charger (needs work)
  - General electrical load with power electronics control
  - through power electronics
- Assume utility grid interface controlled to always draws rated power at rated speed

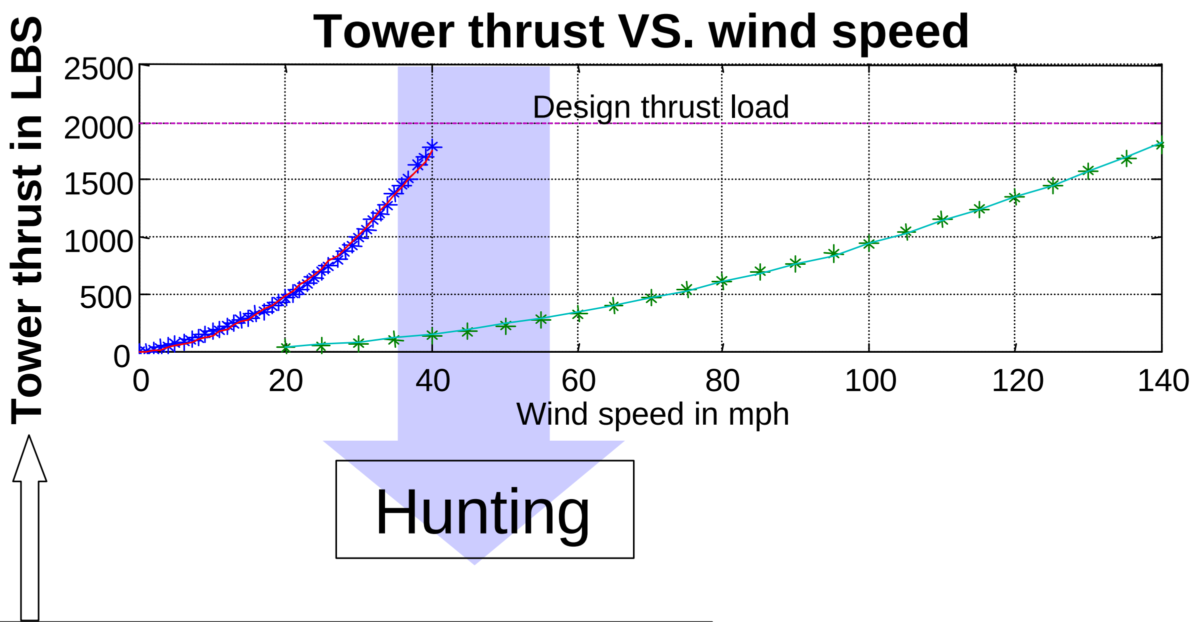
Reduced Efficiency After Furling



After furling, Turbine operates at significantly less efficiency.

In this range of wind speeds, Turbine furls and unfurls constantly. This "hunting" behavior, while consistent with the over-speed protection function, is undesirable.

Hunting Behavior

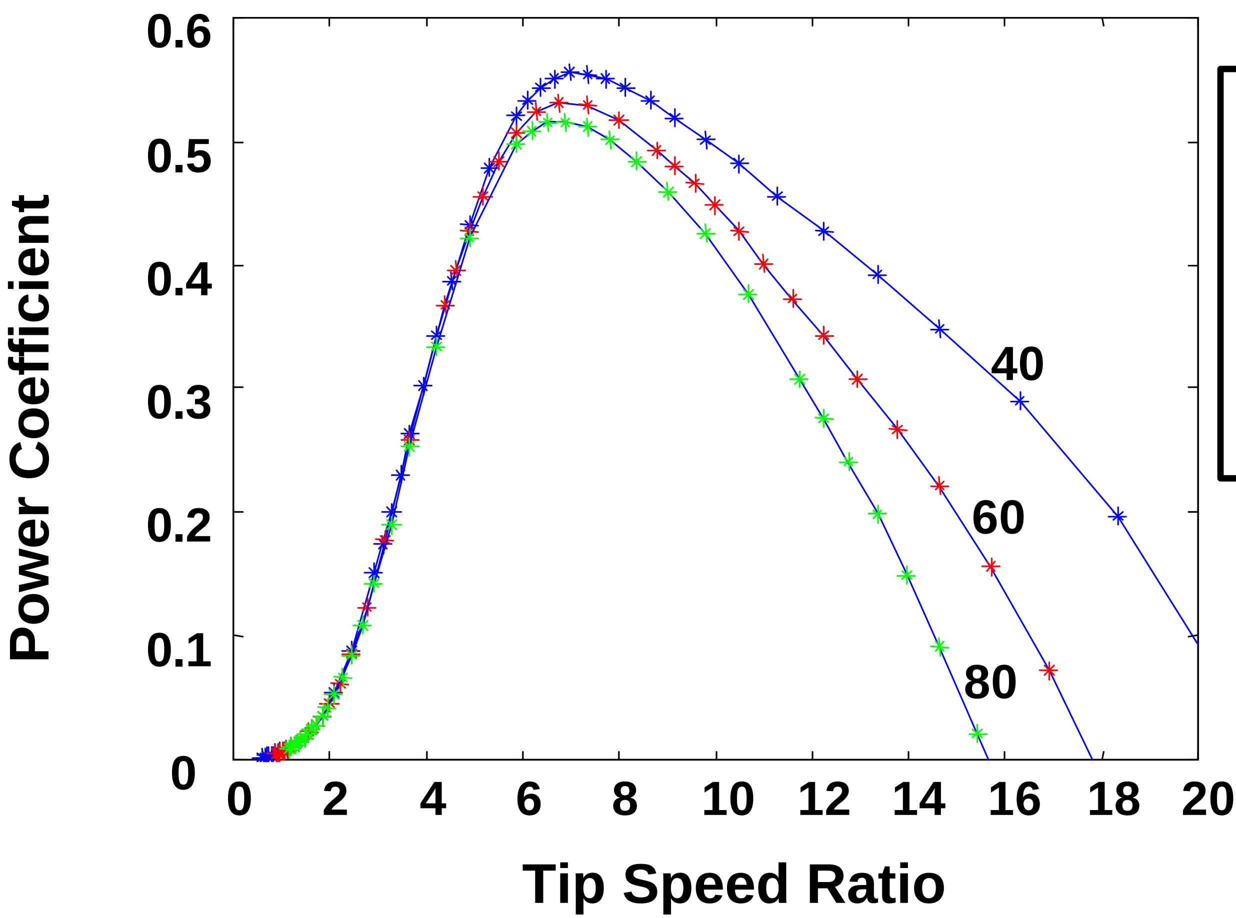


We can use tower thrust experimental data to approximate turbine thrust as a function of wind speed or rpm.

YawDyn Software

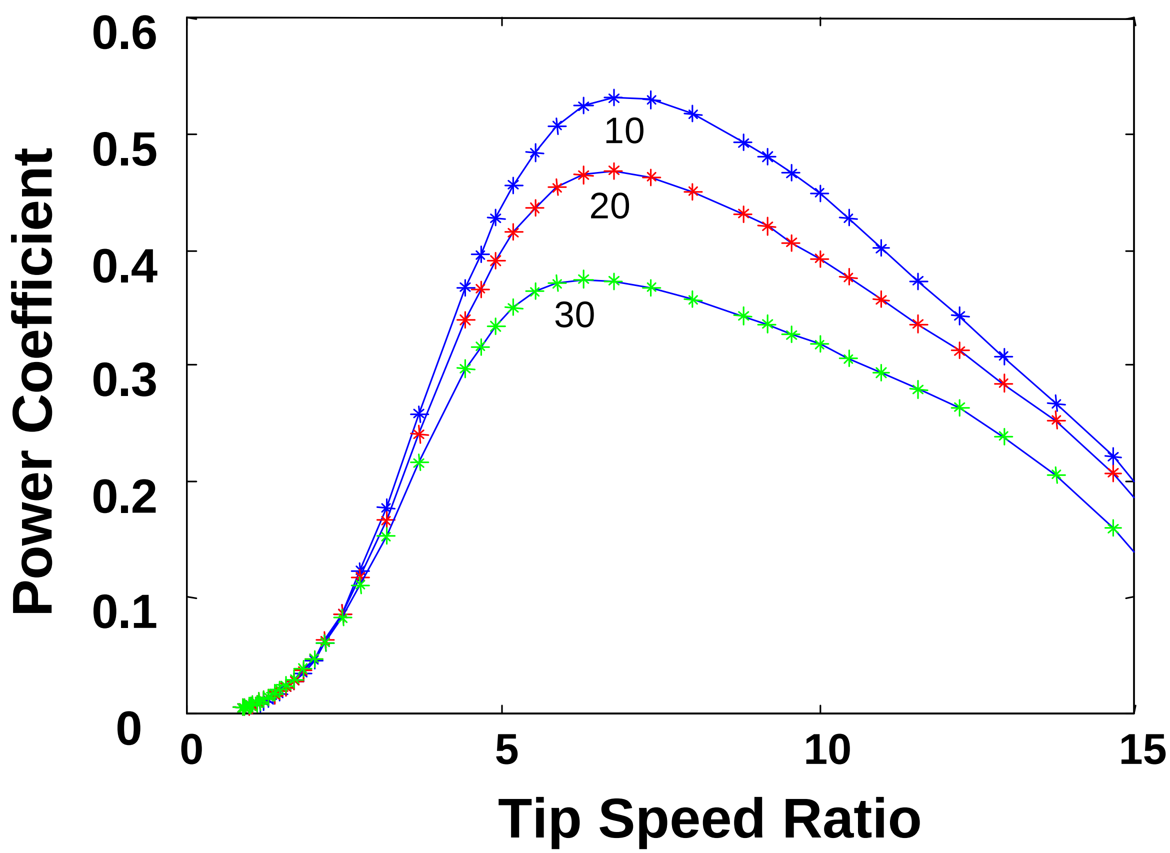
- Complexity of HAWT aerodynamics mathematical models
- Once the geometry of the turbine and blades are specified, the aerodynamic forces and moments of the turbine can be computed using YawDyn.
- YawDyn is a software package developed by C. Hansen and his colleagues at the University of Utah.
- Numerical Computations of Aerodynamic forces expensive numerically.
- Example YawDyn calculation for a 40 kW Bergey-type turbine.

YawDyn Power Curves



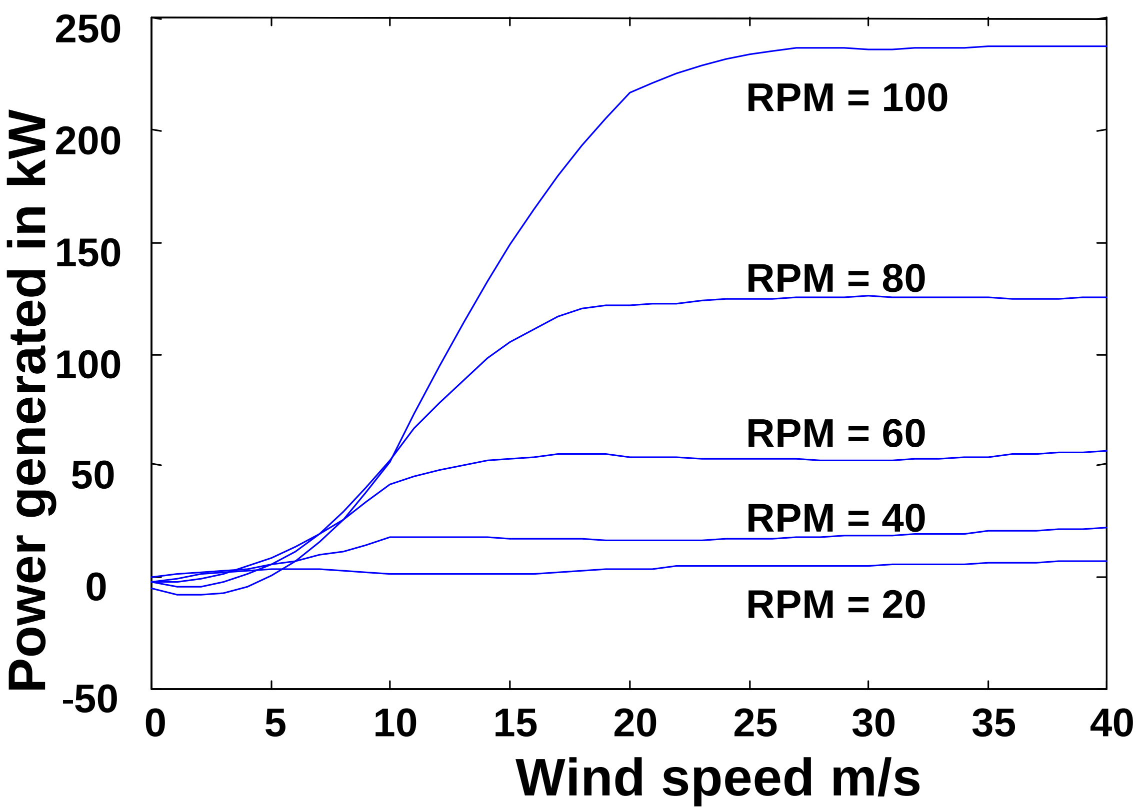
- 40 kW wind turbine
- at 10 degrees wind angle
- different rpms

Different (Yaw -wind) angles



- 40 KW Turbine
- At 60 rpm
- Dtheta = angle between wind and rotor

Power Curves at different rpms

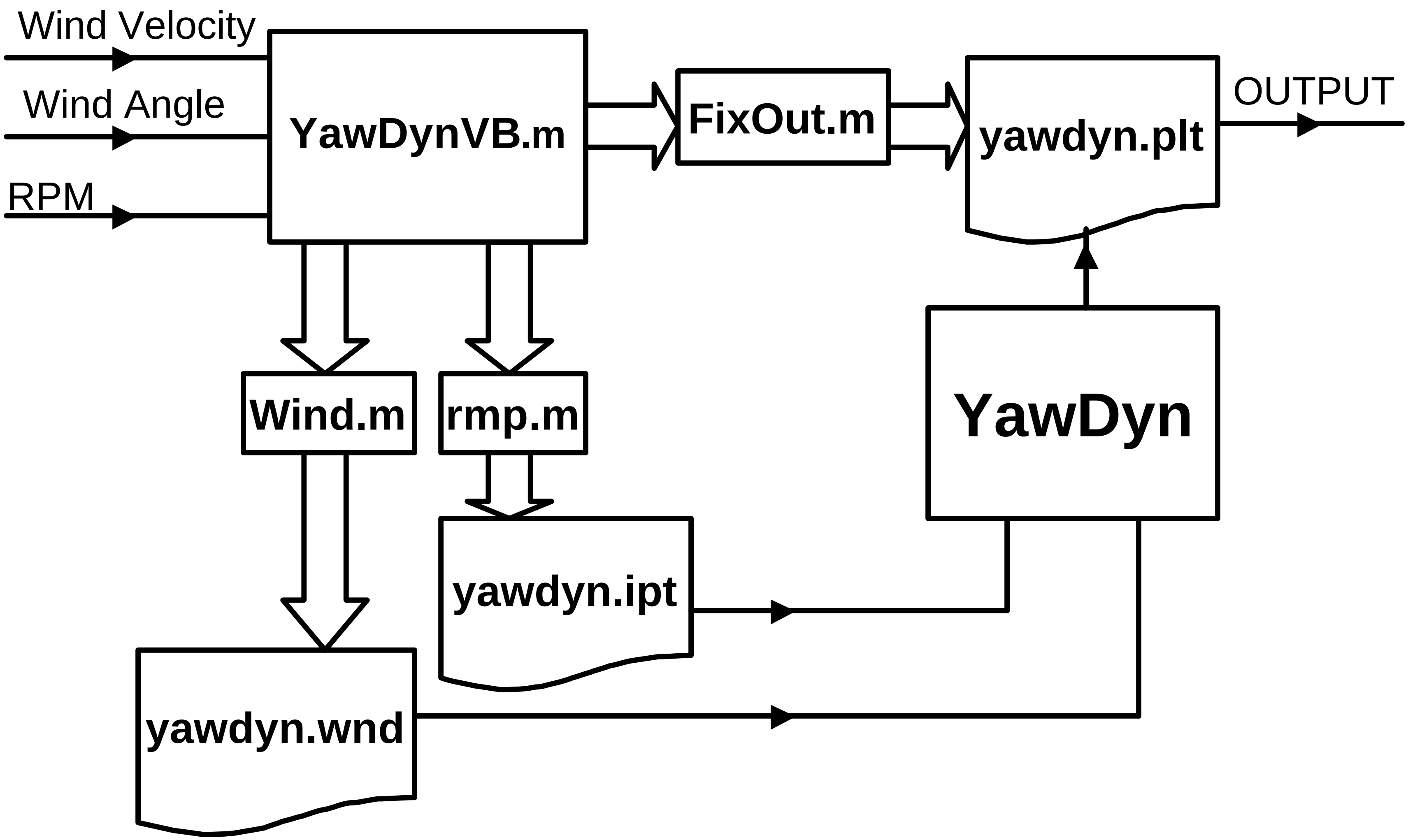


- Power levels off for every rpm as wind speed increases
- Power increases with rpm
- Maximum allowable rpm

Fuzzy Approximation of YawDyn

- YawDyn is computationally expensive for real-time simulations
- Alternative: To generate (learn) a fuzzy inference system (FIS) that approximates YawDyn.
- Gathering a sufficient amount of input/output data to use for the learning process.
- Gathering made possible by a MATLAB interface that allows running several iterations of YawDyn with out user interaction.

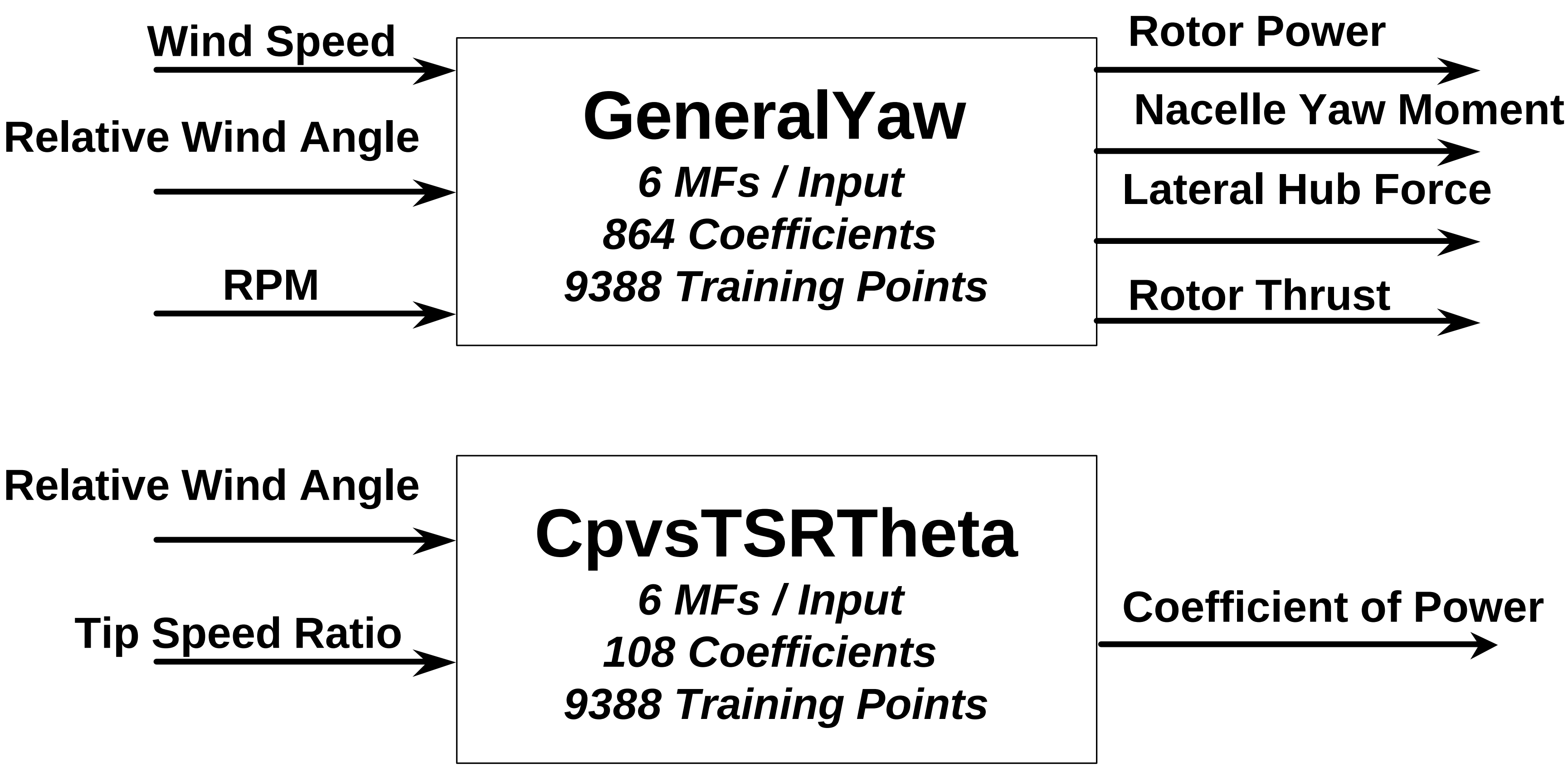
MATLAB Interface



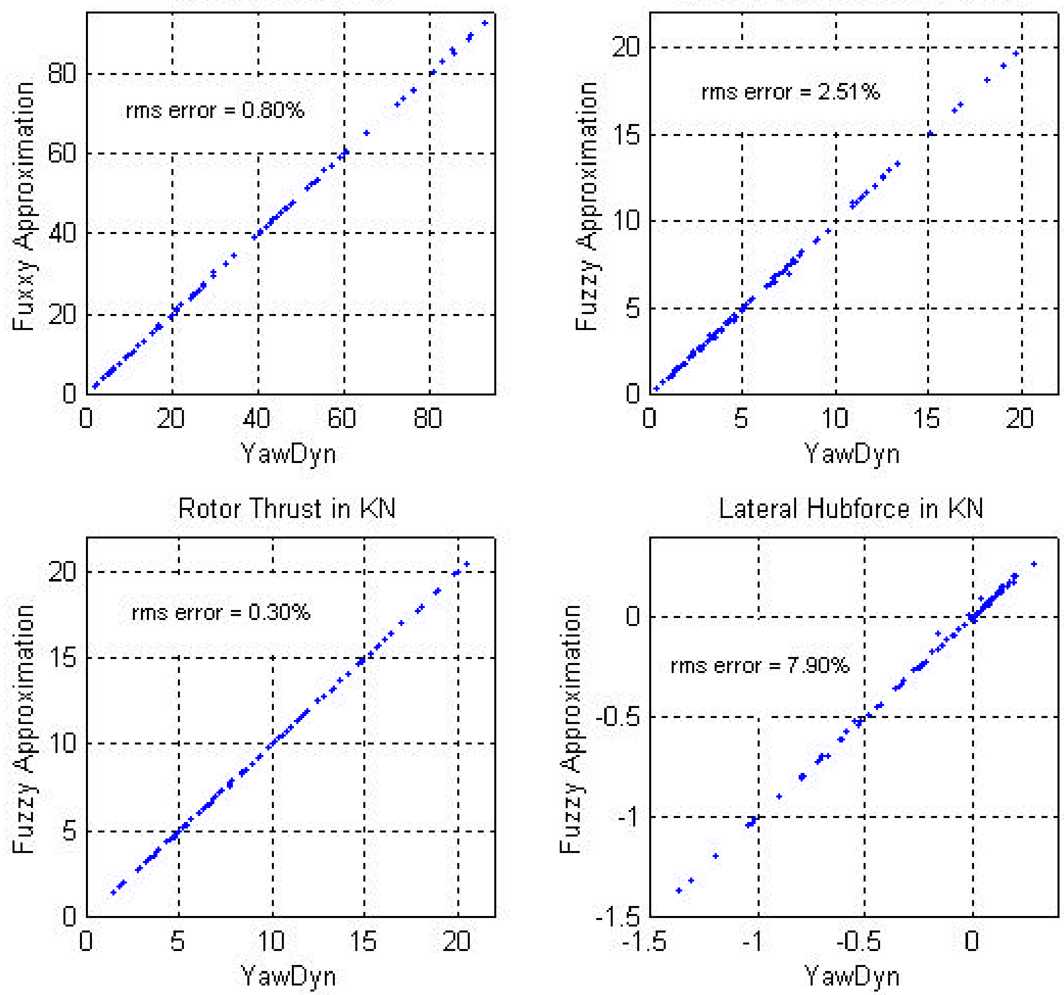
Architecture of Proposed FIS

- Several FIS architectures can be used to approximate YawDyn.
- Direct approach: Use wind speed, relative wind angle, and rpm as inputs.
- Simplest Approach: Use power coefficient—C_P versus the tip speed ratio—TSR.
- Selecting a specific approach includes a trade off between learning time and complexity and approximation quality.
- The next slides will show the implemented FISs.

Two Fuzzy Inference Systems



YawDyn vs. Sugeno GernalYaw



Using random samples

V_{wind}	Δq	rpm	P_{rotor}	$\hat{P}_{rotor}$	*YawDyn* Run Time (sec)	*FIS* Run Time (sec)
35	**31**	**35**	**17.95**	**18.06**	**4.306**	**0.110**
29	**50**	**21**	**6.744**	**6.732**	**3.185**	**0.130**
17	**11**	**34**	**9.825**	**9.848**	**4.427**	**0.100**
29	**18**	**61**	**57.33**	**57.30**	**7.361**	**0.110**
12	**41**	**48**	**21.80**	**22.04**	**7.691**	**0.110**
23	**53**	**82**	**83.08**	**83.37**	**11.28**	**0.130**
18	**17**	**45**	**22.88**	**23.11**	**6.209**	**0.130**

Performance of FIS2

Δq	TSR	P_C	$\widehat{P}_C$	*YawDyn* Run Time (sec)	*FIS* Run Time (sec)
13	**1.2732**	**0.0123**	**0.0129**	**7.400**	**0.0710**
1	**2.7322**	**0.1196**	**0.1209**	**16.35**	**0.0100**
56	**3.2620**	**0.0924**	**0.0929**	**11.71**	**0.0200**
48	**0.7716**	**0.0051**	**0.0053**	**3.485**	**0.0100**
16	**0.9502**	**0.0062**	**0.0059**	**3.905**	**0.0200**
44	**4.8564**	**0.2233**	**0.2155**	**7.351**	**0.0100**
27	**1.0104**	**0.0076**	**0.0076**	**5.398**	**0.0100**

for random test input samples

Fuzzy Linearization

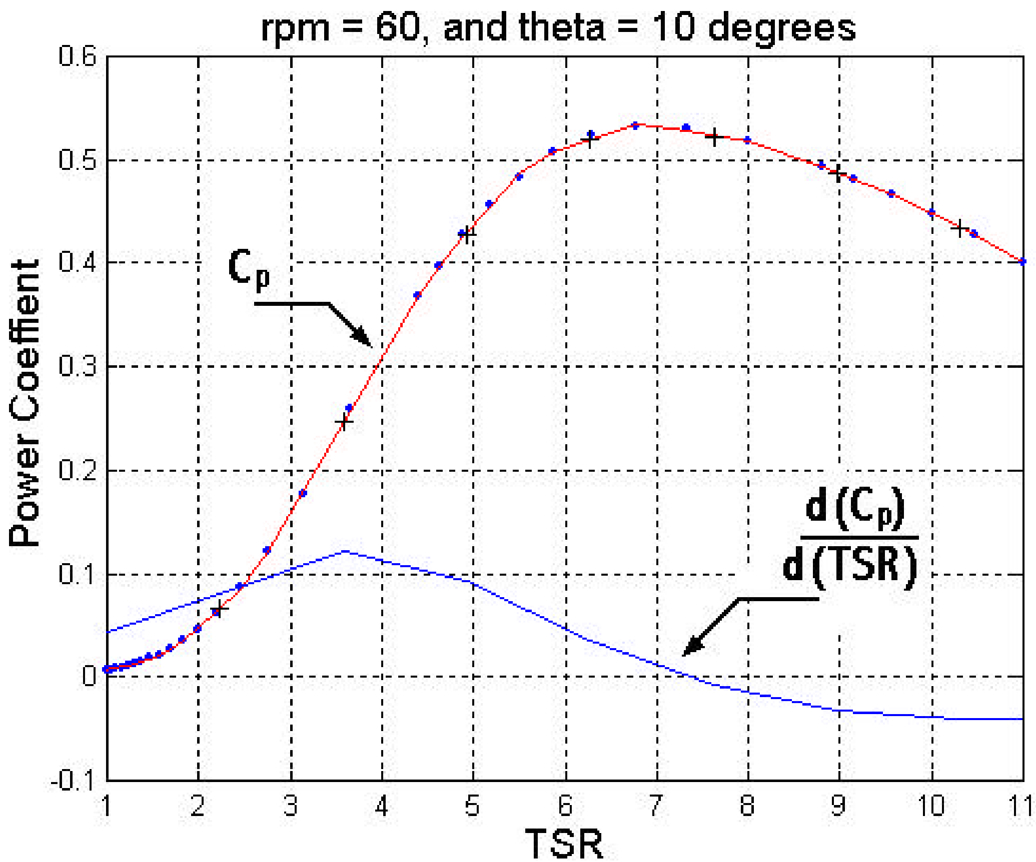
- **Use the Interpretable Sugeno Approximator (ISA)**
- **Make its consequent polynomials rule-centered.**
- **A typical ISA rule has the form:**

$$R^k\text{: If } x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } A_2 \text{ and} \ldots\ x_n \text{ is } A_n \text{ Then}$$

$$u^k = b_0^k + b_1^k(x_1 - r_1^k) + b_2^k(x_2 - r_2^k) + \ldots + b_n^k(x_n - r_n^k)$$

- r^k is the rule center (centers of the membership functions of all inputs tested by the k^{th} rule.)
- If the membership functions are chosen properly (local and differentiable everywhere),
- the b coefficients can be interpreted as Taylor series coefficients.

Interpretation of Coefficients

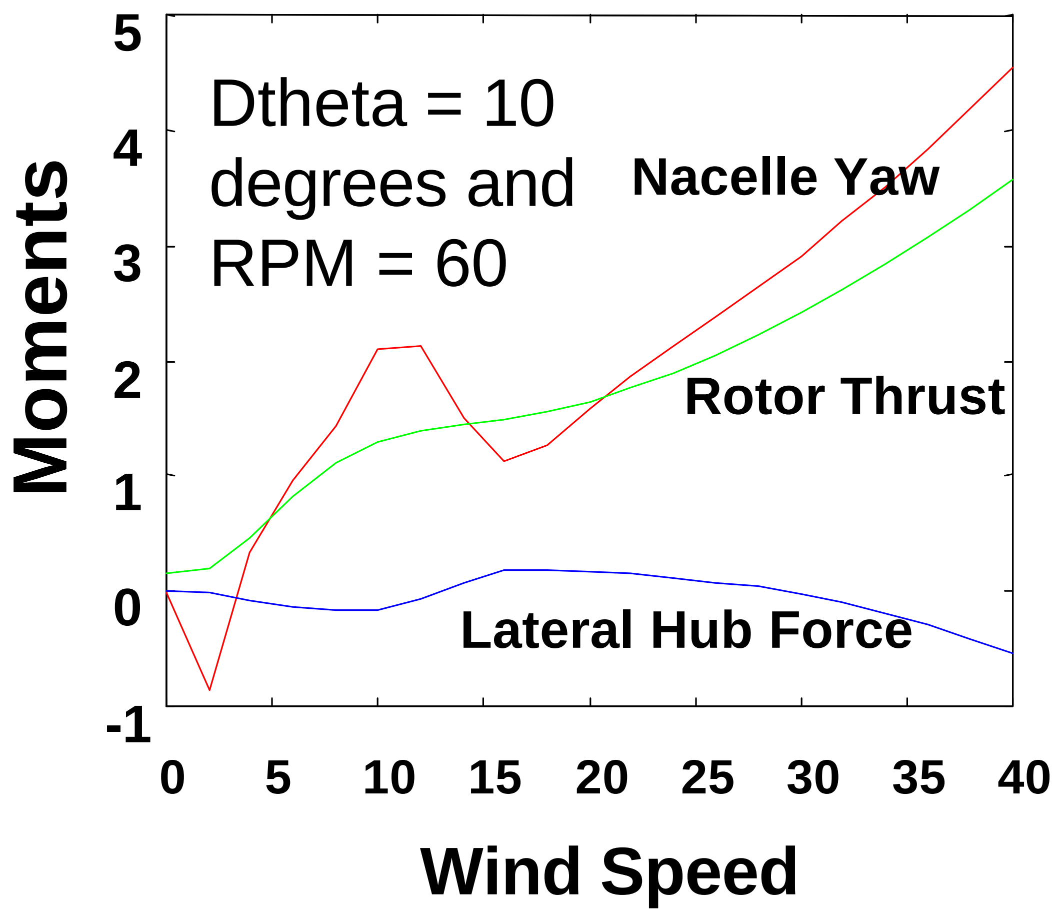


Some coefficients of the Sugeno engine can be interpreted as the derivative of the function at rule centers. In this case, these are aerodynamic sensitivity derivatives.

Conclusions ANNIE 99

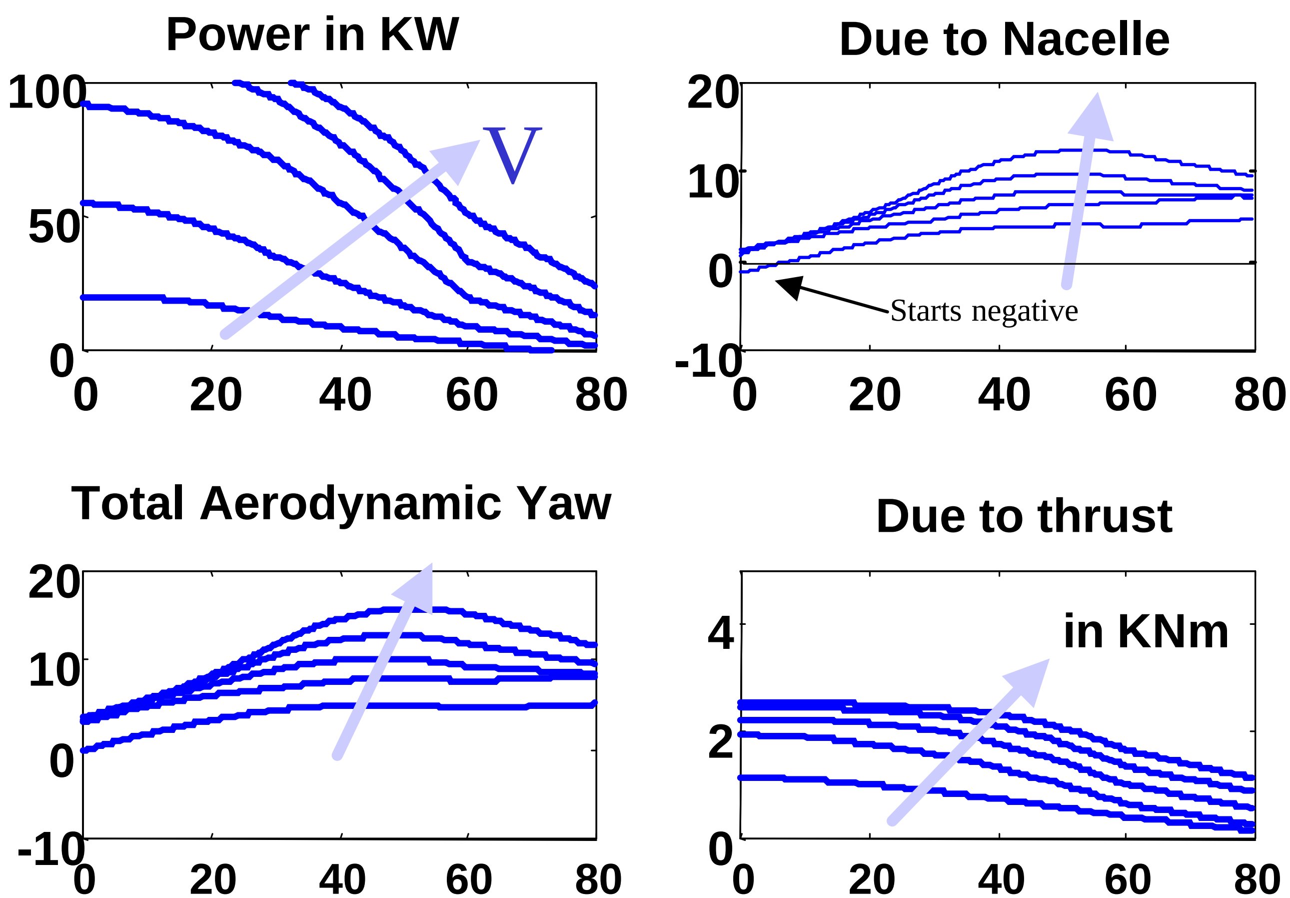
- Fuzzy approximation techniques were used to derive FISs that approximate YawDyn.
- Excellent approximation
- FIS with fewer inputs learns faster
- More general FIS approximates better
- 1 to 2 orders of magnitude speed up in computation
- Using ISA yields estimates of derivatives as well to obtain linearized models

Moment Curves for 40KW



- Nacelle Moment is dominant!
- Hence specifying furling wind speed is not easy

V_{wind} = [7 10 13 16 19] m/sec, 80 rpm



Angle between wind and rotor (degrees)

Yaw Moment of Nacelle becomes dominant as the rotor moves away from wind.

Equilibrium Conditions

$$0 = M_{YawDyn}(V_{wind}, \Delta\theta, \omega) + M^{\theta}_{tail}(\Delta\theta, \psi, V_{wind}) + M^{\theta}_{control}$$

$$\frac{\partial V}{\partial \psi}(\psi) = M^{\psi}_{tail}(V_{wind}, \Delta\theta - \psi) + M_{stop} + M^{\psi}_{control}$$

Generator Torque-Speed Characteristics
(dependent on electrical load and power electronics)

$$\begin{array}{ccc} \Delta\theta, & \psi, & M^{\theta}_{control} \\ \omega, & M_{stop} & M^{\psi}_{control} \end{array}$$

Unknowns

3 Equations

Onset of Furling

- $\psi = 0$ and $M_{stop} = 0$, $M^{\theta}_{control} = 0$
- $M^{\psi}_{control} = 0$ (Yaw moment on yaw axis only)
- $\psi-$condition:

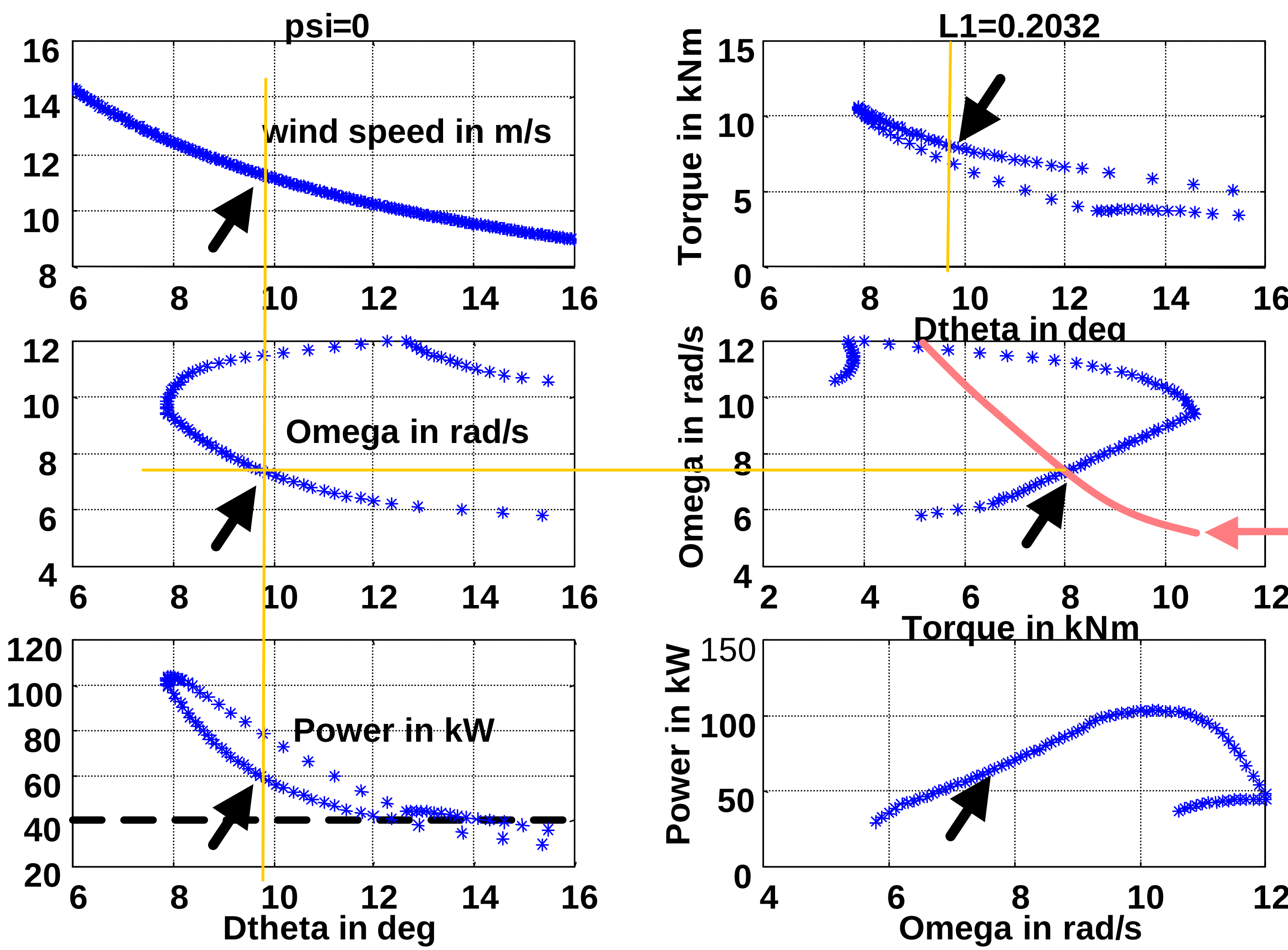
$$V_{furl}(\Delta\theta) = \sqrt{\frac{M_T g L_3 \sin\gamma \cos\beta}{\frac{1}{2}\rho A_{tail} L_{ac} \cos\beta \left(C_L \cos\Delta\theta + C_D \sin\Delta\theta\right)}}$$

- $\theta-$condition:

$$0 = M_{YawDyn}\left(V_{furl}(\Delta\theta), \Delta\theta, \omega\right) + M^{\theta}_{tail}\left(\Delta\theta, 0, V_{furl}(\Delta\theta)\right)$$

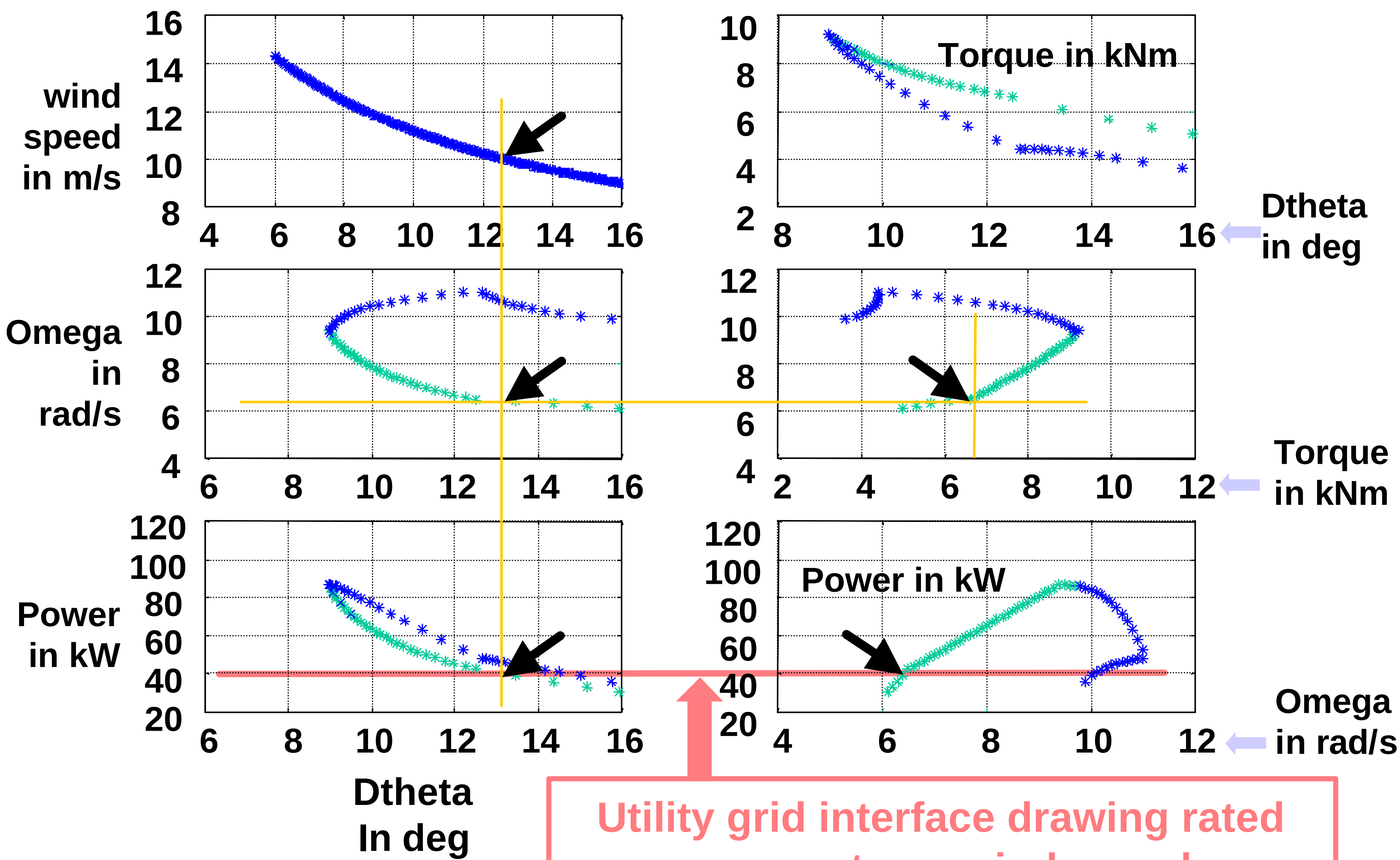
has solutions $\omega(\Delta\theta)$

- From YawDyn: $P\left(V_{furl}(\Delta\theta), \omega(\Delta\theta), \Delta\theta\right) \Rightarrow$ Turbine Torque-Speed Curve



Onset of Furling. No control.

Any generator torque-speed Curve

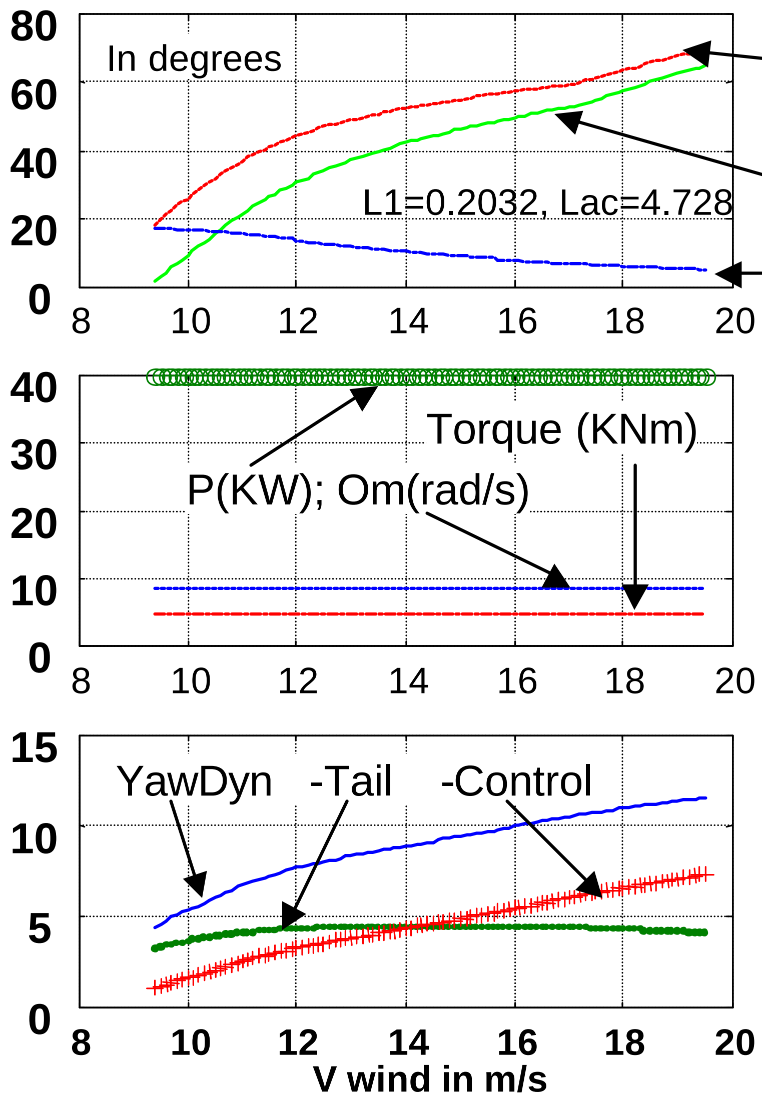


Utility grid interface drawing rated power at any wind speed

Design Yaw Control for constant 40 KW at any wind speed

- Solve for $\Delta\theta\,(V_{wind})$

  Solve $\boxed{P_{YawDyn}\,(V_{wind}, \Delta\theta, \omega_{rated}) = 40KW}$ for $\Delta\theta$
- Assume $M^{\psi}_{control} = 0$ for simplicity. Solve $\boxed{\psi-\text{condition}}$ for $\Delta\theta\,(\psi)$
- From $\boxed{\theta-\text{condition,}}$ find $M^{\theta}_{control}\,(V_{wind})$
- Need to measure V_{wind} to implement control?
- Express as $M^{\theta}_{control}\,(\psi)$ instead? as $M^{\theta}_{control}\,(\omega)$? as $M^{\theta}_{control}$ (Hub Torque)?



Yaw = angle between rotor and wind

Furling = angle between tail and rotor

Yaw-Furling »angle between wind and tail

Furling Curves obtained under the assumption of constant rated (40KW) power delivered to electrical generator under rated speed.

Yaw Moments in KNm.

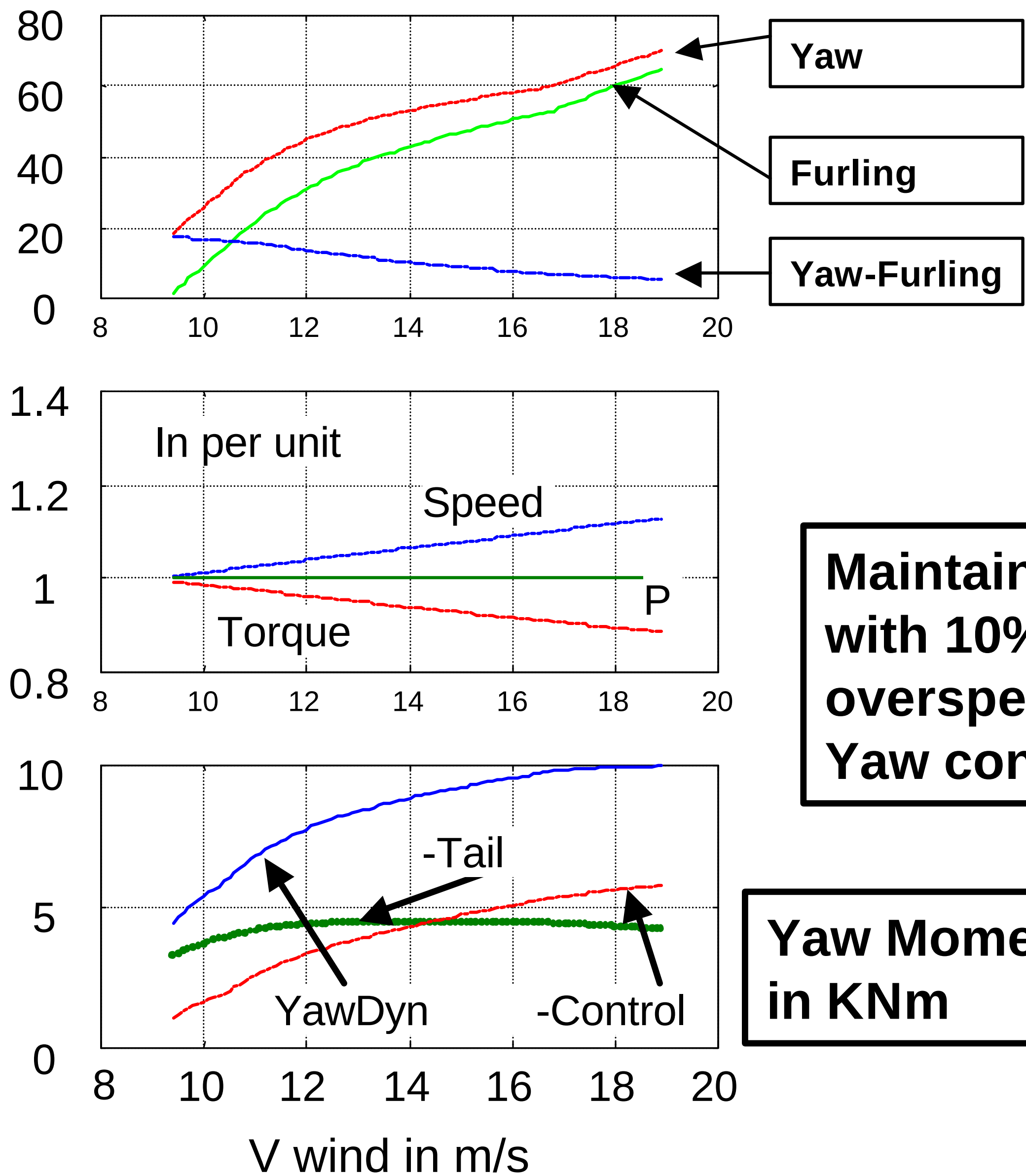
- Tail Moment essentially constant!
- YawDyn: Rotor thrust + Nacelle + Lateral
- Difference must be picked up by Yaw Control Torque (Acting on nacelle only)

Design Yaw Control for any desired schedule of Power and rpm

- Solve for $\Delta\theta\,(V_{wind})$

$$P_{YawDyn}\left(V_{wind}, \Delta\theta, \omega^{desired}\,(V_{wind})\right) = \text{desired } P^{desired}\,(V_{wind})$$

- Solve $\psi-$condition for $\Delta\theta\,(\psi)$
- From $\theta-$condition, find $M^{\theta}_{control}\,(V_{wind})$
- Measure V_{wind}? Express as $M^{\theta}_{control}\,(\psi)$? as ....?



10% allowable Overspeed

Maintaining 40KW output but with 10% allowable overspeed requires 27% less Yaw control

Yaw Moments in KNm

Class of Actuators

$$M^{\psi}_{control} = \rho(\psi)\, M^{\theta}_{control}$$

$$0 = M_{YawDyn}(V_{wind}, \Delta\theta, \omega) + M^{\theta}_{tail}(\Delta\theta, \psi, V_{wind}) + M^{\theta}_{control}$$

$$\frac{\partial V}{\partial \psi}(\psi) = M^{\psi}_{tail}(V_{wind}, \Delta\theta - \psi) + \rho(\psi)\, M^{\theta}_{control}$$

- Assume desired $P^{desired}(V_{wind})$ and $\omega^{desired}(V_{wind})$
- Solve for $\Delta\theta(V_{wind})$

$$P_{YawDyn}\left(V_{wind}, \Delta\theta, \omega^{desired}(V_{wind})\right) = P^{desired}(V_{wind})$$

- Combine and solve for $\psi(V^*)$

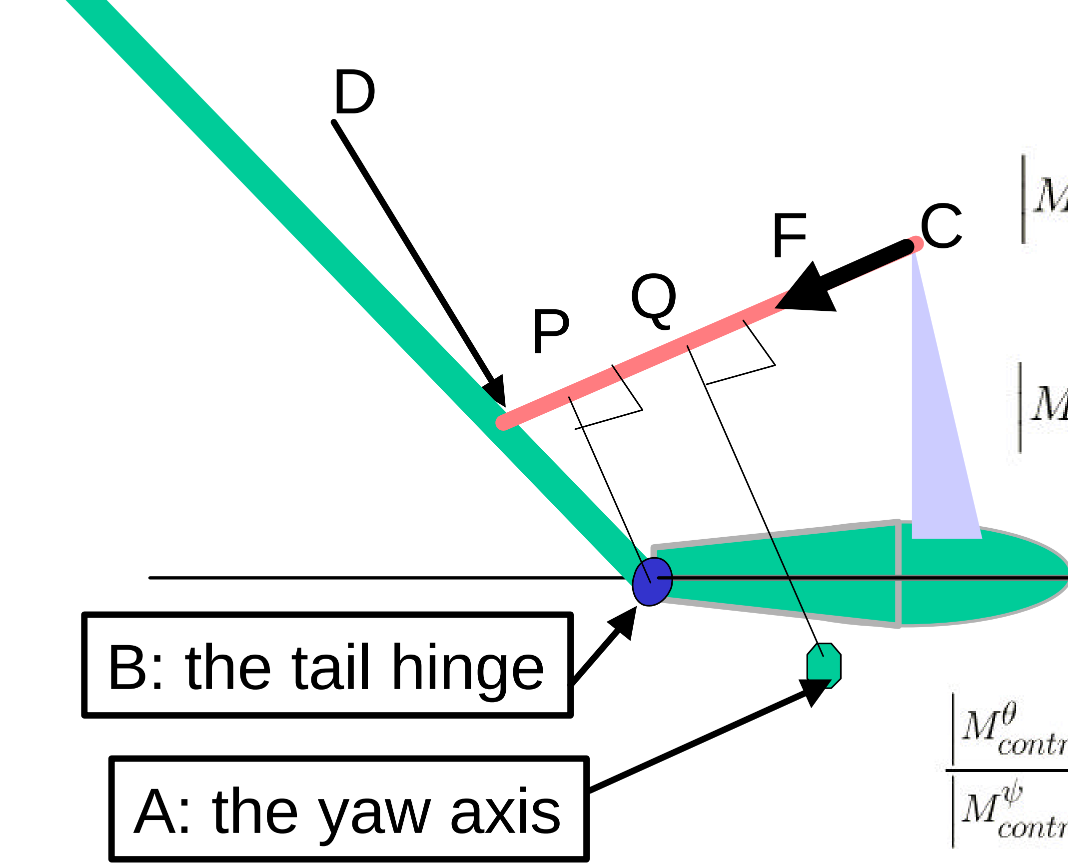
$$0 = \frac{\partial V}{\partial \psi}(\psi) - M^{\psi}_{tail}(V^*, \Delta\theta^* - \psi)$$
$$+\rho(\psi)\, M_{YawDyn}(V^*, \Delta\theta^*, \omega^*) + \rho(\psi)\, M^{\theta}_{tail}(\Delta\theta^*, \psi, V^*)$$

- All quantities are now related.

Linear Motor Geometry

$$\overrightarrow{BD} \times \overrightarrow{DC} = \left(\overrightarrow{BP} + \overrightarrow{PD}\right) \times \overrightarrow{DC} = \overrightarrow{BP} \times \overrightarrow{DC}$$

$$\overrightarrow{AD} \times \overrightarrow{DC} = \left(\overrightarrow{AQ} + \overrightarrow{QD}\right) \times \overrightarrow{DC} = \overrightarrow{AQ} \times \overrightarrow{DC}$$



$$\left|M^{\psi}_{control}\right| = F\left|\overrightarrow{BP}\right| = F\frac{\left|\overrightarrow{BD} \times \overrightarrow{DC}\right|}{\left|\overrightarrow{DC}\right|}$$

$$\left|M^{\theta}_{control}\right| = F\left|\overrightarrow{AQ}\right| = F\frac{\left|\overrightarrow{AD} \times \overrightarrow{DC}\right|}{\left|\overrightarrow{DC}\right|}$$

$$\frac{\left|M^{\theta}_{control}\right|}{\left|M^{\psi}_{control}\right|} = \frac{\left|\overrightarrow{AD} \times \overrightarrow{DC}\right|}{\left|\overrightarrow{BD} \times \overrightarrow{DC}\right|}$$ function of ψ only

Conclusions

- Simulation software in MATLAB and Simulink
- Equations of motion for furling mechanism derived
- Reliance on YawDyn
- Hybrid fuzzy crisp modeling for real-time simulations
- Nacelle moment dominant in furling behavior. Reshape?
- Aerodynamics of tail?
- Generator/Load/Power Electronics? (d-q Simulink model)
- Analysis of equilibrium
  - onset of furling
  - scheduled power/rpm as function of V_{wind}
  - linear motor
- Linearization? Stability? Hopf bifurcation?